Modeling Production and Inventory Problems in Discrete Time

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Outline

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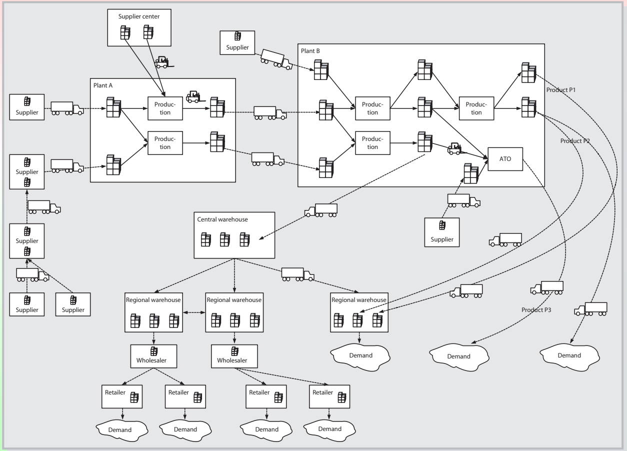
1. Introduction
2. Setups
3. Uncertainty
4. Setups and Uncertainty
5. Conclusion

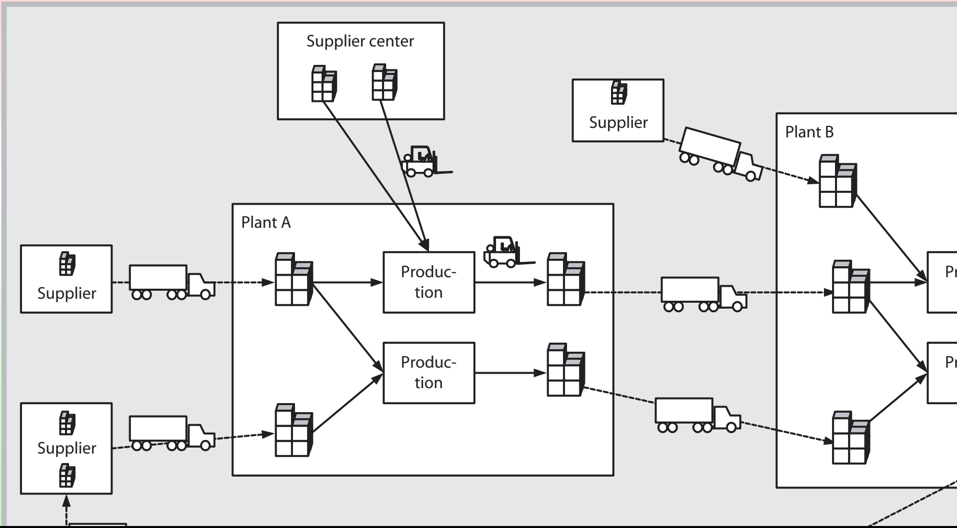
Supply Network

Multi-Product Multi-Level Production and Inventory System



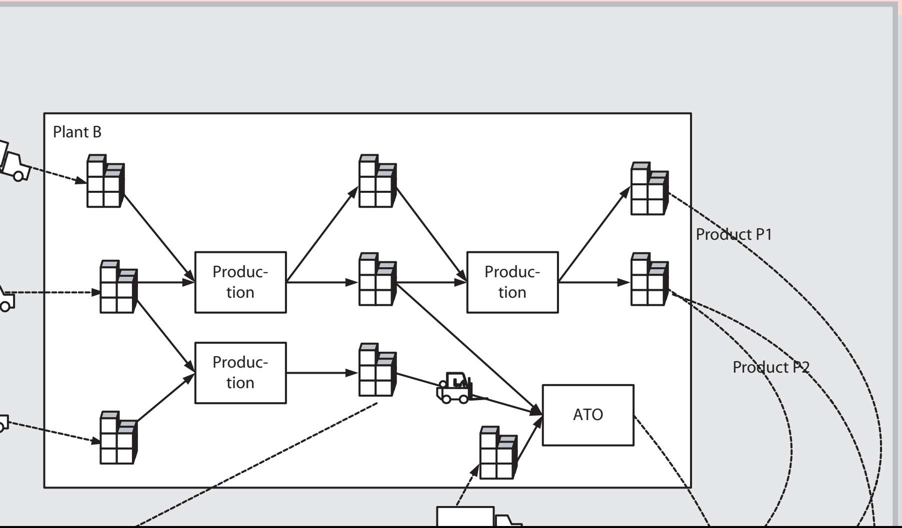
Supply Network

Multi-Product Multi-Level Production and Inventory System



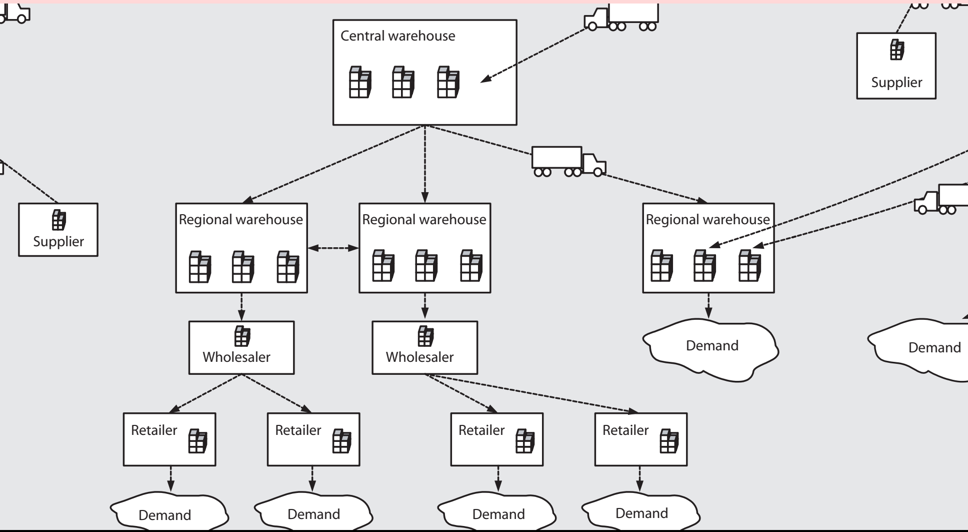
Supply Network

Multi-Product Multi-Level Production and Inventory System



Supply Network

Multi-Product Multi-Level Production and Inventory System



The Planning Environment

Operative Planning in Production and Inventory Systems

What a planner in industry sees ...

- Random {demands, availability of resources, lead times, yield ...}
- Dynamic {demands, capacities}
- Finite capacities, limited resources
- Responsibility for complete logistic processes, not only inventory, handling or transportation

Decision Support

Planning Approaches

How can the planner handle this situation?

- Push approach, use dynamic planning models, forecast future events and treat them as deterministic
- Pull approach, react on observed (random) events

Decision Support

Dynamic Demand

Dynamic demand is usually treated with planning models which consider ...

- Discrete time axis (time buckets: days, weeks)
- Deterministic demand (output of a forecasting procedure)

and which should include ...

- Capacity constraints (lead times are internal)

The planner makes a detailed production plan in advance before the demand occurs. If the model was correct, the plan may be feasible.

Decision Support

Random Demand

Random demand is usually treated with standard inventory models with . . .

- Continuous time axis
- Stationary demand
- No capacity constraints (lead times are external)

The result of an inventory model is a decision rule that defines how to react on demand occurrences.

The Problem

Required: The Integrated View

Industry requires a planning approach which ...

- is aware of dynamic demands
- accounts for uncertainty
- respects capacities (not on the average, but in each period)
- handles setups (lot sizing)

$\Rightarrow$ Dynamic planning + Capacities + Randomness $\Leftarrow$

Setups

The Origin of Lot Sizing

- In the presence of setups (times or costs) lot sizing is required.
- This requires the anticipation of future demands (forecasting).

Evolution of Lotsizing Approaches

The Time Axis in Lotsizing Models

- Continuous time (stationary conditions)
- Discrete time (dynamic conditions)
  - Big bucket models
  - Small bucket models

Lotsizing Models

Big bucket models

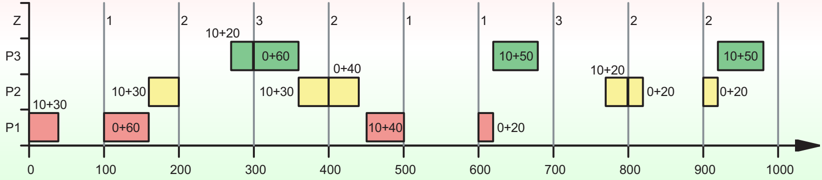
- Single-level models
  - CLSP, CLSP-L
- Multi-level models
  - MLCLSP
  - MLCLSP-L

Small bucket models

- Single-level models
  - CSLP, DLSP, PLSP
- Multi-level models
  - MLPLSP
  - ...

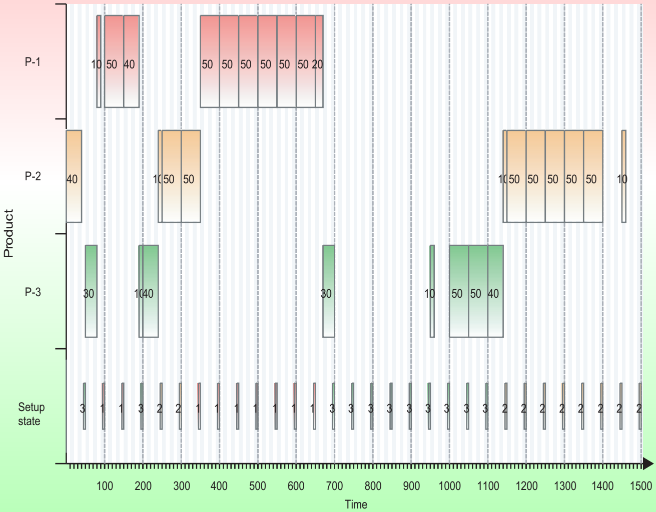
Small Bucket Models

PLSP – Example



Small Bucket Models

PLSP – Example



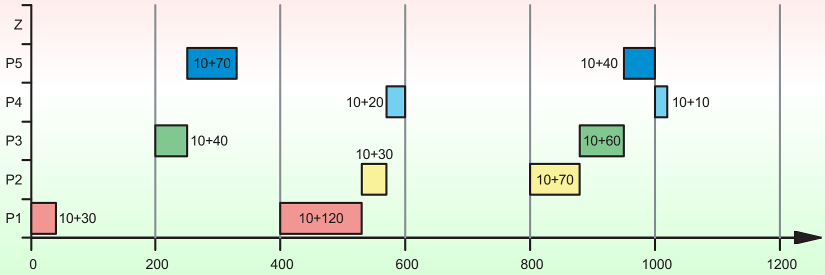
Small Bucket Models

PLSP – Extensions

- Multi-machine PLSP with common setup operator
- Variations
  - Setup time may be larger than a bucket length
  - Minimum lot size
  - Maximum lot size
  - Sequence-dependent setup times/costs
  - Parallel machines
- Useful in deterministic short-term planning situations

Big Bucket Models

CLSP – Example



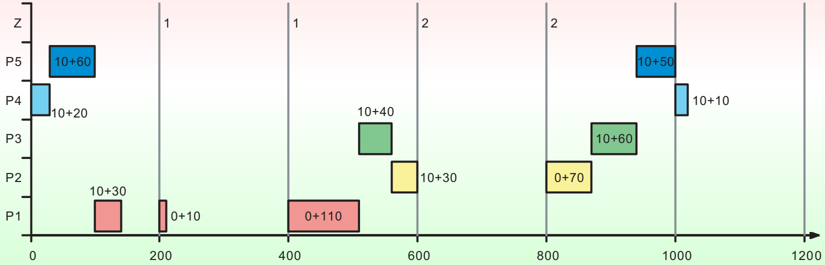
Big Bucket Models

CLSP – Extensions

- Setup carry-over (CLSP-L)
- Parallel machines
- Consideration of random demand

Big Bucket Models

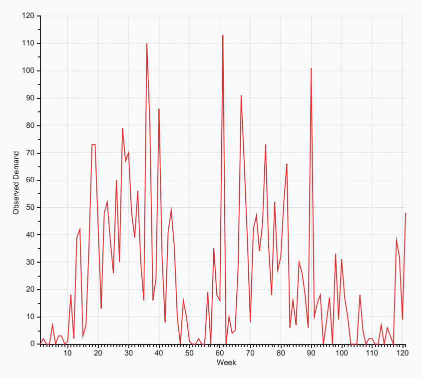
CLSP-L – Example

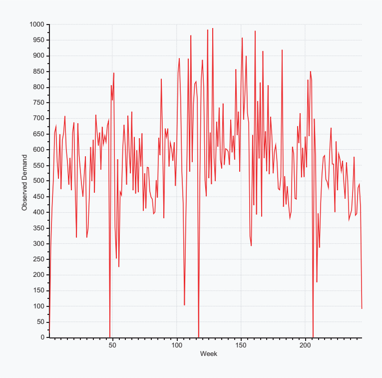


Causes of Uncertainty

- Demand (mean, variation, time of ordering)
- Breakdowns
- Production quality problems
- Delays
- Random setup times
- Planning errors (capacity restrictions)
- Data problems

Observed Weekly Demand





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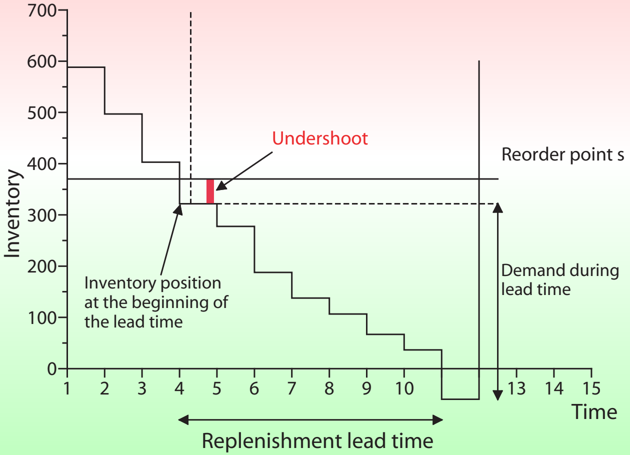
Inventory Models Applied in Industry

(s, q)-Policy in MRP/AP Systems

- The formulas used for the (s, q)-policy are based on a continuous time axis (demand arrivals, review)
  Continuous review: at the beginning of the replenishment time the inventory position is exactly equal to s
- (s, q)-policy with continuous review is unrealistic in industry
- Undershoot

(s, q)-Policy in Discrete Time

Undershoot



(s, q)-Policy in Discrete Time

The Relevance of the Undershoot: Example data

- Time unit: 1 day
- Gamma distributed period demands: $\mu_D = 50$, $\sigma_D = 25$
- Target service level: $\beta = 0.95$
- Lot size: $q = 100$

- Modeling alternatives
  1. Continuous review
  2. The undershoot is directly approximated and accounted for
  3. Continuous review with a lead time inflated by one period

(s, q)-Policy in Discrete Time

The Relevance of the Undershoot: Results

	Calculation of Reorder Point s		
Lead time ℓ	❶ Continuous	❷ With Undershoot	❸ With $\ell + 1$
1	64.25	109.41	128.4
2	128.4	170.86	189.82
3	189.82	230.75	249.72
4	249.72 ($\beta = 0.86$)	289.62	308.59 ($\beta = 0.98$)
	$\beta < 0.95$	$\beta = 0.95$	$\beta > 0.95$

Dynamic and Random Demand

Planning situation

- Product-specific
  - Demand (forecasted averages and variations)

Period t	25/2009	26/2009	...	35/2009
μ_t	...	...	...	...
σ_t	...	...	...	...

  - Holding costs
  - Setup costs
  - Service level
- Period capacities

Dynamic and Random Demand

Alternatives

- Common sense approach (MRP, APS)
  Compute safety stocks and add to forecasted demand
- (s_t, q_t)-policy, (r_t, S_t)-policy
  Use a stationary inventory policy with dynamic adjustment of parameters
- "Static-dynamic uncertainty" strategy
  Fix replenishment periods in advance, adjust production quantity
- "Static uncertainty" strategy
  Fix replenishment periods and quantities in advance

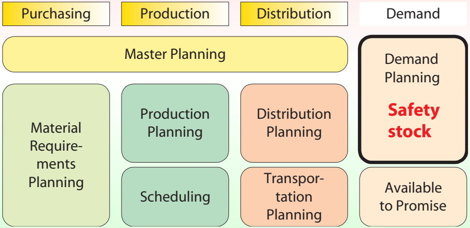
Common sense

MRP, APS

Procedure

- Forecast demand
- Add safety stock to demand
- Determine production schedule

APS Planning Matrix



Problems

- How to compute safety stock
- Safety stock calculation associated to demand planning

Dynamic parameter adjustment

Inventory policies

Procedure

- Forecast demand during lead time
- Determine order size or order cycle in advance (EOQ, ...)
- Observe demand and react (launch a replenishment order)

Problems

- Separation of cycle stock and safety stock
- (s, q): Random timing of production
- (r, S): Random production quantities
- No consideration of capacities

Static-Dynamic Uncertainty Strategy

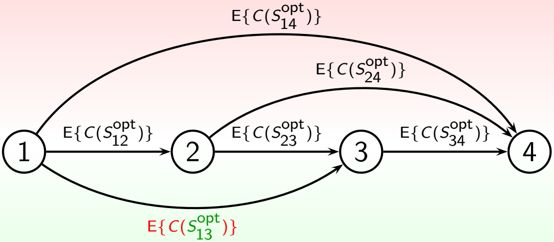
Example

t	μ_t	σ_t	S_{tj}^{opt}	On Hand	Backorders	$ß_c$
1	200	60	336	136.24	0.24	1.00
2	50	15	–	88.31	2.08	0.99
3	100	30	–	20.99	32.67	0.90
4	300	90	746.48	446.48	0	1.00
5	150	45	–	296.53	0.05	1.00
6	200	60	–	109.99	13.46	0.98
7	100	30	–	46.51	36.51	0.93
8	50	15	–	26.47	29.97	0.90
9	200	60	340.35	140.55	0.2	1.00
10	150	45	–	25.34	34.79	0.90

$$E\{Costs\} = 4337$$

Static-Dynamic Uncertainty Strategy

Exact Solution Method



$\mathrm{E}\{C(S_{13}^{\mathrm{opt}})\} =$ Expected cost, if the demand from periods 1 to 2 is available at the beginning of period 1. S_{13}^{opt} is the minimum order level required to reach the target β service level.

Solution method: Shortest-path algorithm

Static-Dynamic Uncertainty Strategy

Pros and Cons

+

- Effect of order size on risk absorbtion included
- Easy coordination of replenishment events for multiple items
- Optimal solution

–

- Random lot sizes
- Random capacity requirements

Static Uncertainty Strategy

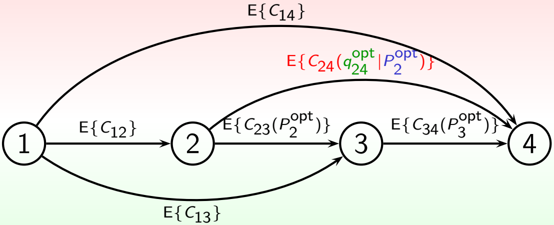
Example

t	μ_t	σ_t	q_{tj}	On Hand	Backorders	ß$_c$
1	200	60	335.47	135.72	0.25	1.00
2	50	15	–	87.83	2.11	0.99
3	100	30	–	20.77	32.94	0.90
4	300	90	770.62	456.09	0.00	1.00
5	150	45	–	306.32	0.23	1.00
6	200	60	–	122.97	16.65	0.97
7	100	30	–	58.59	35.62	0.93
8	50	15	–	36.59	28.00	0.90
9	200	60	461.19	222.54	5.26	0.97
10	150	45	–	102.58	30.04	0.90

$E\{Costs\} = 4549$

Static Uncertainty Strategy

Exact Solution Method



$E\{C_{24}(q_{24}^{opt}|P_2^{opt})\} =$ Expected cost, if the demand from periods 2 to 3 is available at the beginning of period 2. q_{24}^{opt} is the minimum replenishment quantity required to reach the target β service level.
P_2^{opt} ist the optimum path to node 2.

Static Uncertainty Strategy

Heuristic Solution Method

Dynamic lot sizing heuristics with adjusted criterion, e. g. Silver-Meal criterion

$$E\{C_{\tau t}\} = \frac{s + h \cdot \sum_{\ell=\tau}^{t} E\left\{\left[I_{\tau-1}(P_{\tau-1}) + q^*_{\tau t} - \sum_{i=\tau}^{\ell} D_i\right]^+\right\}}{t - \tau + 1}$$

Static Uncertainty Strategy

Waiting Time Distribution

Probability distribution of the waiting times: $P\{W_c \leq 1\} \geq 0.98$

t	μ_t	σ_t	q_{tj}	ß$_c$
1	200	60	301.34	0.99
2	50	15	–	0.97
3	100	30	–	0.83
4	300	90	914.06	1.00
5	150	45	–	1.00
6	200	60	–	0.99
7	100	30	–	0.98
8	50	15	–	0.96
9	200	60	330.45	0.96
10	150	45	–	0.88

w	$P\{W_{c1} <= w\}$
0	0.8333
1	0.9799
2	0.9968
3	1.0000
w	$P\{W_{c4} <= w\}$
0	0.9636
1	0.9801
2	0.9958
3	1.0000
w	$P\{W_{c9} <= w\}$
0	0.8773
1	0.9799
2	1.0000

Static Uncertainty Strategy

Pros and Cons

+

- Effect of order size on risk absorbtion included
- Optimal solution
- Deterministic capacity requirements

−

- Costs

Dynamic and Random Demand with Finite Capacities

Available Solution Approaches

The Real Challenge: Finite Capacities

- Limited number of solution approaches
- Based on Static Uncertainty Strategy (fixed lot sizes)
- Backorder costs (per item and period)

Dynamic and Random Demand with Finite Capacities

A New Heuristic: ABC_β

ABC_β Heuristic: Basic Principle

Transform the matrix of demands into a matrix of production quantities

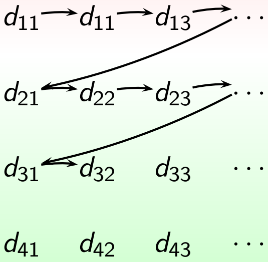
Ingredients of the Heuristic

- A Sequence of products (Costs, ...),
- B Optimality criterion (LUC, Silver-Meal, ...),
- C Visiting sequence of demand matrix cells (E, S, SE)

Dynamic and Random Demand with Finite Capacities

ABC$_\beta$ Heuristic: Visiting Sequence

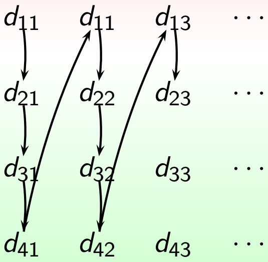
East



Dynamic and Random Demand, Finite Capacities

ABC$_\beta$ Heuristic: Visiting Sequence

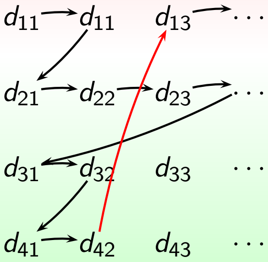
South



Dynamic and Random Demand, Finite Capacities

ABC$_\beta$ Heuristic: Visiting Sequence

South-East



Dynamic and Random Demand, Finite Capacities

ABC_β Heuristic: Lot Sizing Criteria

Adjusted versions of

1. Least period cost criterion
2. Least unit cost criterion
3. Least total cost criterion
4. Absolute cost criterion

ABC$_\beta$ Heuristic: Example

Optimum Solutions of Single-Item Problems with Infinite Capacities

Product $k=1$				Product $k=2$					
t	μ_D	σ_D	Lot sizes	t	μ_D	σ_D	Lot sizes	Workload	Capacity
1	110	11	107.84	1	48	5	47.06	154.9	160
2	49	5	56.83	2	75	8	112.24	169.07	160
3	0	0	0	3	15	2	0	0	160
4	82	8	116.04	4	10	1	0	116.04	160
5	40	4	0	5	15	2	0	0	120
6	65	7	73.34	6	70	7	76.01	149.35	120

ABC$_\beta$ Heuristic: Example

Finite versus Infinite Capacities

	Infinite capacities			Finite capacities		
	Lot sizes			Lot sizes		
t	$k=1$	$k=2$	Workload	$k=1$	$k=2$	Workload
1	107.84	47.06	154.9	107.931	47.097	155.028
2	56.83	112.24	169.07	56.871	89.231	146.102
3	0	0	0	0	0	0
4	116.04	0	116.04	80.02	22.368	102.388
5	0	0	0	47.425	17.599	65.024
6	73.34	76.01	149.35	61.987	58.013	120

This is the End . . .

- Gap between theory and practice
- Lot sizing has made much progress
- Inclusion of random demand requires a unifying time scale
- Discrete time scale offers the opportunity to evaluate any setup pattern w. r. t. service levels
- Service levels should match the requirements of industrial planners