Dynamic uncapacitated lot sizing with random demand under a fillrate constraint

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Agenda

Planning situation

Dynamic and Random Demand

- Demand (forecasted averages and variations)

Period t	25/2009	26/2009	...	35/2009
μ_t	...	...	...	...
σ_t	...	...	...	...

- Holding costs
- Setup costs
- Service level

Alternatives

- Common sense approach (MRP, APS)
  Compute safety stocks and add to forecasted demand
- (s_t, q_t)-policy, (r_t, S_t)-policy
  Use a stationary inventory policy with dynamic adjustment of parameters
- "Static-dynamic uncertainty" strategy
  Fix replenishment periods in advance, adjust production quantity
- "Static uncertainty" strategy
  Fix replenishment periods and quantities in advance

Model Formulation I

Model SSIULSP$^q_{\beta_c}$:

$$\text{Minimize } Z = \sum_{t=1}^{T} \left(s \cdot \gamma_t + h \cdot E\left\{[I_t]^+\right\} \right) \tag{1}$$

s. t.

$$I_{t-1} + q_t - D_t = I_t \qquad t = 1, 2, \ldots, T \tag{2}$$

$$q_t - M \cdot \gamma_t \leq 0 \qquad t = 1, 2, \ldots, T \tag{3}$$

$$I_t^{f,\text{prod}} = -\left[I_{t-1} + q_t\right]^- \qquad t = 1, 2, \ldots, T \tag{4}$$

$$I_t^{f,\text{end}} = -\left[I_t\right]^- \qquad t = 1, 2, \ldots, T \tag{5}$$

$$F_t = I_t^{f,\text{end}} - I_t^{f,\text{prod}} \qquad t = 1, 2, \ldots, T \tag{6}$$

Model Formulation II

$$I_t = (I_{t-1} + 1) \cdot (1 - \gamma_t) \qquad t = 1, 2, \ldots, T \qquad (7)$$

$$I_0 = -1 \qquad (8)$$

$$\omega_t = \gamma_{t+1} \qquad t = 1, 2, \ldots, T-1 \qquad (9)$$

$$\omega_T = 1 \qquad (10)$$

$$1 - \frac{E\left\{\sum_{j=t-I_t}^{t} F_j\right\}}{E\left\{\sum_{j=t-I_t}^{t} D_j\right\}} \geq \beta_c^\star \qquad t \in \{t \mid \omega_t = 1\} \qquad (11)$$

Symbols used I

$\beta_c^\star$	target fillrate
D_t	demand in period t (random variable)
F_t	backorder in period t (random variable)
γ_t	binary setup indicator in period t
h	inventory holding cost
I_t	net inventory at the end of period t (random variable)
$I_t^{f,\text{end}}$	backlog at the end of period t (random variable)
$I_t^{f,\text{prod}}$	backlog immediately after production in period t (random variable)
l_t	number of periods since the last setup prior to period t
M	large number

Symbols used II

ω_t	indicator variable: $\omega_t = 1$, if production takes place in period $t+1$; $\omega_t = 0$, otherwise
q_t	production quantity in period t
s	setup cost
T	length of planning horizon
$[x]^+$	$= \max\{0, x\}$
$[x]^-$	$= \min\{0, x\}$

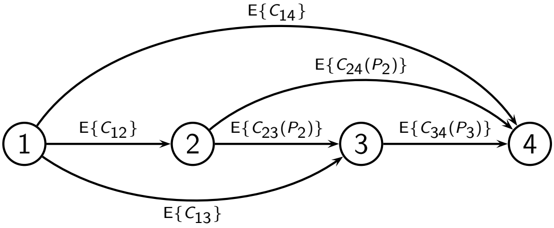
Expected Inventory

$$E\{I_t^p\} = \int_0^{Q^{(t)}} (Q^{(t)} - y) \cdot f_{Y^{(t)}}(y) \cdot dy$$
$$= Q^{(t)} - E\{Y^{(t)}\} + G^1_{Y^{(t)}}(Q^{(t)}) \qquad t = 1, 2, \ldots \quad (14)$$

$Q^{(t)}$ – cumulated production quantity from period 0 to t
$Y^{(t)}$ – cumulated demand from period 0 to t

1. Introduction
   - The Problem
   - Solution Approaches
2. Optimization Model
   - Formulation
3. **Solution approaches**
   - Exact solution
   - Heuristic solution
4. Numerical Results
   - Experiment 1
   - Experiment 2
5. Conclusion

Shortest-Path Network



$$E\{C_{\tau t}\} = s + h \cdot \sum_{\ell=\tau}^{t-1} E\left\{\left[I_{\tau-1}(P_\tau) + q^{\star}_{\tau t} - \sum_{i=\tau}^{\ell} D_i\right]^+\right\} \qquad (15)$$

Shortest-Path Network

Setup in period					On hand inventory
1	2	3	4	5	$E\{I_5^p\}$
1	0	0	0	0	79.61
1	1	0	0	0	81.14
1	1	1	0	0	87.01
1	1	1	1	0	103.89
1	1	1	1	1	118.95
1	0	1	0	0	86.95
⋮	⋮	⋮	⋮	⋮	⋮

Table: Expected on-hand inventory at the end of period 5 as a function of the setup pattern

Solution Procedure

$\mathcal{M} := \mathcal{U} := \{2, 3, \ldots, T\}$
$\mathcal{E} := \{(\tau, t) | \tau = 1, 2, \ldots, T; t = \tau + 1, \tau + 2, \ldots, T - 1\}$
for all $((0, t) \in \mathcal{E}$ with $t \in \mathcal{U})$ **do**
    Predecessor(t):= 1; $C(t) := \mathrm{E}\{C_{1t}\}$
end for
while $(\mathcal{M} \neq \emptyset)$ **do**
    Select $\tau \in \mathcal{M}$ with minimum $C(\tau)$
    $\mathcal{M} := \mathcal{M} \setminus \tau; \mathcal{U} := \mathcal{U} \setminus \tau$
    if $(\tau = T)$ **then**
        end
    else
        for all $((\tau, t) \in \mathcal{E}$ with $t \in \mathcal{U})$ **do**
            $\mathcal{M} := \mathcal{M} \cup t$
            if $(C(\tau) + \mathrm{E}\{C_{\tau t}\} < C(t))$ **then**
                Predecessor(t):= τ
                $C(t) := C(\tau) + \mathrm{E}\{C_{\tau t}\}$
            end if
        end for
    end if
end while

Dynamic Lot Sizing Heuristic

$\tau := 1$
while $(\tau < T)$ **do**
    $t := \tau$
    while $(t < T)$ **do**
        if $(C_{\tau t} \leq C_{\tau,t+1})$ **then**
            $t := t + 1$
        else
            Make current lotsize for period τ permanent.
            $\tau := t + 1$
        end if
    end while
end while

Silver-Meal Rule

$$E\{C_{\tau t}\} = \frac{s + h \cdot \sum_{\ell=\tau}^{t} E\left\{\left[I_{\tau-1}(P_{\tau-1}) + q^*_{\tau t} - \sum_{i=\tau}^{\ell} D_i\right]^+\right\}}{t - \tau + 1} \tag{16}$$

Least-Unit-Cost rule

$$E\{C_{\tau t}\} = E\left\{\frac{s + h \cdot \sum_{\ell=\tau}^{t}\left[I_{\tau-1}(P_{\tau-1}) + q^*_{\tau t} - \sum_{i=\tau}^{\ell} D_i\right]^+}{\sum_{i=\tau}^{t} D_i}\right\} \quad (17)$$

Least-Total-Cost rule

$$E\{C_{\tau t}\} = E\left\{s + h \cdot \sum_{\ell=\tau}^{t}\left[I_{\tau-1}(P_{\tau-1}) + q^*_{\tau t} - \sum_{i=\tau}^{\ell} D_i\right]^+\right\} \quad (18)$$

Expected Demands

Series #	$E\{D_t\}$											
1	92	92	92	92	92	92	92	92	92	92	92	93
2	80	100	125	100	50	50	100	125	125	100	50	100
3	50	80	180	80	0	0	180	150	10	100	180	95
4	10	10	15	20	70	180	250	270	230	40	0	10

Parameters

Series #	T	s	TBO	CV_D	$\beta_c^\star$
1	12	500	1–12	$\{0.1, 0.2, 0.3, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
2	12	500	1–12	$\{0.1, 0.2, 0.3, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
3	12	500	1–12	$\{0.1, 0.2, 0.3, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
4	12	500	1–12	$\{0.1, 0.2, 0.3, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$

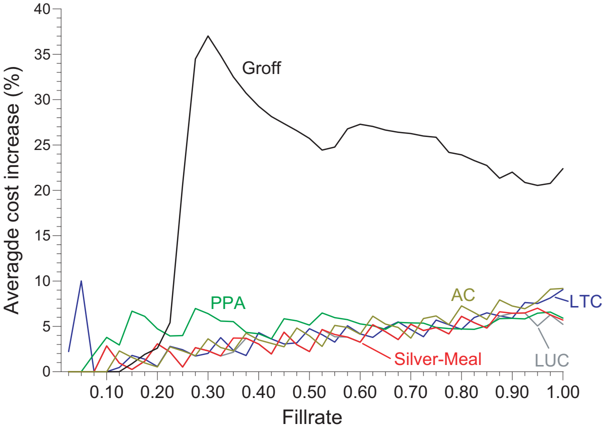
Results

Series #	Heuristic	Average cost increase (%)	Maximum cost increase (%)	% Optimal	% Worst
1	LUC	4.9	30.4	56.6	3.76
	SM	5.0	30.4	56.0	3.97
	LTC	5.3	29.1	48.6	7.00
	PPA	5.8	43.5	57.8	19.23
	AC	6.1	30.1	48.6	7.52
	Groff	24.5	52.7	3.9	75.24
2	SM	5.9	32.7	42.8	4.79
	LTC	7.2	28.7	41.5	5.31
	LUC	7.5	31.3	39.9	5.83
	PPA	7.9	49.9	51.4	17.50
	AC	7.9	32.9	38.3	5.31
	Groff	24.8	60.2	0.4	69.90

Results

Series #	Heuristic	Average cost increase (%)	Maximum cost increase (%)	% Optimal	% Worst
3	AC	9.7	37.0	35.4	3.02
	PPA	10.2	55.8	46.0	3.96
	SM	10.7	36.6	29.5	1.04
	LTC	11.5	41.1	33.2	5.83
	Groff	23.2	58.9	10.3	33.13
	LUC	29.0	67.9	1.8	54.69
4	SM	6.0	30.9	49.9	1.88
	AC	13.2	50.3	27.8	0.73
	PPA	15.3	59.0	34.2	14.58
	LTC	15.8	52.3	22.5	2.19
	Groff	17.1	47.6	14.3	7.81
	LUC	39.1	61.2	3.4	74.48

Results for Demand Series 1



Parameters

$E\{D_t\} \simeq \text{Uniform}(0, 100)$

Series #	T	s	TBO	CV_D	$\beta_c^\star$
5	5	500	1–5	$\{0.1, \ldots, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
6	10	500	1–5	$\{0.1, \ldots, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
7	15	500	1–15	$\{0.1, \ldots, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$
8	20	500	1–15	$\{0.1, \ldots, 0.4\}$	$\{0.5, 0.525, 0.05, \ldots, 0.975\}$

Results

Series #	Heuristic	Average cost increase (%)	Maximum cost increase (%)	% Optimal	% Worst
5	SM	4.7	35.4	65.4	14.47
	AC	5.6	39.7	61.2	13.07
	LTC	5.9	63.4	62.6	11.16
	LUC	10.5	64.4	50.6	25.25
	Groff	13.8	60.5	34.7	36.77
	PPA	15.1	56.7	38.6	42.72
6	SM	4.2	32.7	43.7	3.75
	AC	10.2	41.2	21.6	10.05
	LTC	11.5	44.7	16.5	13.35
	Groff	13.8	70.5	13.9	27.95
	LUC	15.4	55.3	12.6	21.25
	PPA	19.9	60.4	12.2	43.78

Results

Series #	Heuristic	Average cost increase (%)	Maximum cost increase (%)	% Optimal	% Worst
7	LTC	8.5	42.8	40.4	3.14
	AC	8.6	44.8	39.5	2.26
	PPA	9.1	64.9	53.1	15.04
	LUC	11.0	53.9	35.3	8.18
	SM	12.1	40.3	25.6	5.96
	Groff	30.3	64.4	2.3	68.47
8	SM	8.8	38.8	21.1	1.54
	AC	10.7	46.5	17.9	1.48
	LTC	11.6	46.2	17.0	3.64
	PPA	12.5	66.2	32.5	16.42
	LUC	13.9	49.4	14.7	9.33
	Groff	29.9	64.6	1.0	69.41

Conclusion

- Exact solution for the stochastic Wagner-Whitin problem
- Adjusted cost criteria used in standard dynamic lot sizing heuristics
- Silver-Meal rule superior to Groff rule
- Directly applicable in ERP/AP systems
- Static uncertainty strategy: no nervousness, no bullwhip effect
- Target service level (instead of backorder costs)
- Possible extension: Capacities (done)